Introduction

A beam can fail by reaching the plastic moment and becoming fully plastic (see last section) or fail prematurely by:

1. LTB, either elastically or inelastically
2. FLB, either elastically or inelastically
3. WLB, either elastically or inelastically

If the maximum bending stress is less than the proportional limit when buckling occurs, the failure is elastic. Else it is inelastic.

For bending $\phi_b M_n (\phi_b = 0.9)$

TABLE User Note F1.1
Selection Table for the Application of Chapter F Sections

| Section in | Cross Section | Flange Slenderness | Web Slenderness | Limit States |
| Chapter F  |              |                   |                 |              |
| F2         |               |                   |                 | Y, LTB       |
| F3         |               | NG, S             | C               | LTB, FLB     |
| F4         |               | C, NC, S          | C               | Y, LTB, FLB, TFY |
| F5         |               | C, NC, S          | S               | Y, LTB, FLB, TFY |
Lateral Torsional Buckling (LTB)

- Compact Members (AISC F2)
- Failure Mode
- Plastic LTB (Yielding)
- Inelastic LTB
- Elastic LTB
- Moment Gradient Factor $C_b$

Lateral Torsional Buckling (cont.)

- Nominal Flexural Strength $M_n$
  - Plastic when $L_b \leq L_p$ and $M_a = M_p$
  - Inelastic when $L_p < L_b \leq L_r$ and $M_p > M_a \geq M_r$
  - Elastic when $L_b > L_r$ and $M_a < M_r$

Failure Mode

A beam can buckle in a lateral-torsional mode when the bending moment exceeds the critical moment.

I-Beam in a Buckled Position
Lateral Torsional Buckling (cont.)

**Elastic LTB**
- coupled differential equations for rotation and lateral translation
  \[ M_z = GJ \frac{d\phi}{dz} - EC_w \frac{d^2\phi}{dz^2} \] (8.5.10)
  
  where
  - \( M_z \) = moment at location \( z \) along member axis
  - \( z \) = axis along member length
  - \( \phi \) = angle of twist
  - \( G \) = shear modulus
  - \( J \) = torsional constant (AISC Table 1-1 for torsional prop.)
  - \( E \) = modulus of elasticity
  - \( C_w \) = warping constant (AISC Table 1-1 for warping)

Lateral Torsional Buckling (cont.)

**Plastic LTB (Yielding)**
- Flexural Strength
  \[ M_a = M_p = F_y Z \] (AISC F2-1)
  
  where \( Z \) = plastic section modulus & \( F_y \) = section yield stress

- **Limits**
  - Lateral bracing limit
    \[ L_b < L_p = 1.76r_b \sqrt{\frac{E}{F_y}} \] (AISC F2-5)
  - Flange and Web width/thickness limit (AISC Table B4.1)
  (Note: \( L_{pd} \) in Salmon & Johnson Eq. (9.6.2) is removed from AISC 13th Ed.)

Lateral Torsional Buckling (cont.)

**Inelastic LTB** \( L_p < L_b \leq L_r \)
- Flexure Strength (straight line interpolation)
  \[ M_a = C_b \left( M_p - \left( M_p - M_r \right) \left( \frac{L_p - L_b}{L_r - L_p} \right) \right) \leq M_p \] (9.6.4)

  or

  \[ M_a = C_s \left( M_p - 0.7F_y S_y \left( \frac{L_p - L_b}{L_r - L_p} \right) \right) \leq M_p \] (AISC F2-2)

Lateral Torsional Buckling (cont.)

**Elastic LTB** \( L_b > L_r \)
- Flexure Strength
  \[ M_a = F_y S_y L \leq M_p \] (AISC F2-3)

  \[ F_y = \frac{E}{(L_b/S_y)^2 \left( \frac{L_b}{S_y} \right)^{1 + 0.078 \frac{Jc}{S_y h_b}} \left( \frac{L_b}{S_y} \right)^{1 + 6.76 \frac{0.7F_y S_y h_b}{E Jc}} \} \] (AISC F2-4)

  (The square root term may be conservatively taken equal to 1.0)

  (c in AISC F2-8a,b for doubly symmetric I-shape, and channel, respectively)

- **Limit**
  \[ L_r = 1.95r_n \frac{E}{0.7F_y} \left( \frac{Jc}{S_y h_b} \right)^{1 + \left( 1 + 6.76 \frac{0.7F_y S_y h_b}{E Jc} \right)^2} \] (AISC F2-6)

  \[ r_n^2 = \frac{\sqrt{L_r C_o}}{S_y} \] (AISC F2-7)
Lateral Torsional Buckling (cont.)

- Moment Gradient Factor $C_b$
  - The moment gradient factor $C_b$ accounts for the variation of moment along the beam length between bracing points. Its value is highest, $C_b=1$, when the moment diagram is uniform between adjacent bracing points.
  - When the moment diagram is not uniform
    \[
    C_b = \frac{12.5 M_{\max}}{2.5 M_{\max} + 3 M_A + 4 M_B + 3 M_C}
    \]  
    \[\text{(AISC F1-1)}\]
    where
    - $M_{\max} = \text{absolute value of maximum moment in unbraced length}$
    - $M_A, M_B, M_C = \text{absolute moment values at one-quarter, one-half, and three-quarter points of unbraced length}$

$C_b$ for a Simple Span Bridge

- Case 1: Laterally braced at ends; points 1 and 5 only; $M_{\max}$ at 3
  - $C_b = 1.14$
- Case 2: Laterally braced at ends and midspan; points 1, 2, 3, and 5 only; $M_{\max}$ at 3
  - $C_b = 1.30$
- Case 3: Laterally braced at end and 1st quarter point; bracing at points 1 and 2; $M_{\max}$ at 3
  - $C_b = 1.52$
- Case 4: Laterally braced at 1st and 2nd quarter points; bracing at points 2 and 3; $M_{\max}$ at 3
  - $C_b = 1.03$
- Case 5: Laterally braced at 1st and 3rd quarter points; bracing at points 2 and 4; $M_{\max}$ at 3
  - $C_b = 1.00$

*Values from 1986 LRFD, Eq. 9.6.12 shown in parenthesis.

Nominal Moment Strength $M_u$ as affected by $C_b$

Flange Local Buckling (FLB)

- Compact Web and Noncompact/Slender Flanges (AISC F3)
- Failure Mode
- Noncompact Flange
- Slender Flange
- Nominal Flexural strength, $M_n = \text{Min} \ (F2, F3)$
Flange Local Buckling (cont.)

- **Failure Mode**
  The compression flange of a beam can buckle locally when the bending stress in the flange exceeds the critical stress.

Flange Local Buckling (cont.)

- **Nominal Flexural Strength** \( M_n \)
  - plastic when \( \frac{b_f}{2t_f} \leq \lambda_p \) and \( M_n = M_p \)
  - inelastic when \( \lambda_p \leq \frac{b_f}{2t_f} \leq \lambda_r \) and \( M_p > M_n \geq M_r \)
  - elastic when \( \frac{b_f}{2t_f} > \lambda_r \) and \( M_n < M_r \)

Flange Local Buckling (cont.)

- **Noncompact Flange** (straight line interpolation)
  - **Flexure Strength**
    \[ M_n = M_p - (M_p - 0.7F_p S_k) \left( \frac{\lambda - \lambda_{pf}}{\lambda_{pf} - \lambda_{pf}} \right) \]  
    *(AISC F3-1)*

Flange Local Buckling (cont.)

- **Slender Flange**
  - **Flexure Strength**
    \[ M_n = \frac{0.9E_k S_c}{\lambda^2} \]  
    *(AISC F3-2)*

\[ k_c = \frac{4}{\sqrt{h/t_w}} \]  
*(kc shall not be less than 0.35 and not greater than 0.76)*

- **Limit** *(AISC Table B4.1)*
Web Local Buckling (WLB)

- Compact or Noncompact Webs (AISC F4)
- Failure Mode
- Compact Web (Yielding)
- Noncompact Web
- Slender Web
- Nominal Flexural Strength, \( M_n = \min (\text{compression flange yielding, LTB, compression FLB, tension flange yielding}) \)

Web Local Buckling (cont.)

- Failure Mode
  The web of a beam can also buckle locally when the bending stress in the web exceeds the critical stress.

Web Local Buckling (cont.)

- Nominal Flexural Strength \( M_n \)
  - plastic when \( \lambda \leq \lambda_r \) and \( M_n = M_p \)
  - inelastic when \( \lambda_r < \lambda \leq \lambda_c \) and \( M_p > M_n \geq M_r \)
  - elastic when \( \lambda > \lambda_c \) and \( M_n < M_r \)

Web Local Buckling (cont.)

- Compression Flange Yielding
  - Flexural Strength
    \[
    M_n = R_{pc} M_{yc} = R_{pc} F_y S_{yc} \quad \text{(AISC F4-1)}
    \]
    where \( R_{pc} \) = web plasticification factor (AISC F4-9a, b) & \( F_y \) = section yield stress
  - Limits (AISC Tables B4.1)
    \[
    L_0 < L_p = 1.1 \sqrt{\frac{E}{F_y}}
    \]
**Web Local Buckling (cont.)**

- **LTB (Inelastic)** \(L_p < L_t \leq L_r\)
  - **Flexure Strength**
    \[
    M_x = C_3 \left[ R_{p_y} M_{y_p} - \left( R_{p_y} M_{y_p} - F_y S_{w} \right) \left( \frac{\lambda - \lambda_{eff}}{\lambda_{eff} - \lambda_{pf}} \right) \right] \leq M_p
    \]
    (AISC F4-12)
    where \(F_L\) = a stress determined by AISC F4-6a, b

- **LTB (Elastic)** \(L_p > L_r\)
  - **Flexure Strength**
    \[
    M_n = F_{ct} S_{xc} \leq R_{p_y} M_{y_c}
    \]
    (AISC F4-3)
    \[
    F_{ct} = \frac{C_2 \pi^2 E}{(l_p/r_f)^2 \left( 1 + 0.078 \frac{J}{S_h b_p \left( l_p \right)^2} \right)}
    \]
    (AISC F4-5)
  - **Limit** (AISC Table B4.1)
    \[
    L_c = \frac{1.95 R_{pt} E}{F_L} \left( \frac{J}{S_h b_p} \right) \left[ 1 + 6.76 \left( \frac{F_y S_{w}}{E J} \right) \right]^2
    \]
    (AISC F4-8)

**Web Local Buckling (cont.)**

- **Compression FLB (Noncompact Flange)**
  - **Flexure Strength**
    \[
    M_x = \left[ R_{p_y} M_{y_p} - \left( R_{p_y} M_{y_p} - F_y S_{w} \right) \left( \frac{\lambda - \lambda_{eff}}{\lambda_{eff} - \lambda_{pf}} \right) \right] \leq M_p
    \]
    (AISC F4-12)

- **Compression FLB (Slender Flange)**
  - **Flexure Strength**
    \[
    M_x = \frac{0.9 E k_x S_{w}}{\lambda^2}
    \]
    (AISC F4-13)
    \[
    k_x = \frac{4}{\sqrt{h / t_c}}
    \]
    (k_x shall not be less than 0.35 and not greater than 0.76)

- **Tension Flange Yielding** \(S_{st} < S_{xc}\)
  - **Flexure Strength**
    \[
    M_n = R_{pt} M_{y_t} = R_{pt} F_y S_{st}
    \]
    (AISC F4-14)
    \[
    R_{pt} = \text{web plastification factor to the tension flange yielding limit}
    \]
    (AISC F4-15a)
    (AISC F4-15b)
Shear Strength

- Failure Mode
- Shear-Buckling Coefficient
- Elastic Shear Strength
- Inelastic Shear Strength
- Plastic Shear Strength

For shear $\varphi_v V_n (\varphi_v = 0.9$ except certain rolled I-beam $h/t_w \leq 2.24\sqrt{E/F_y}$, $\varphi_v = 1.0$)

$$V_n = 0.6F_y A_w C_v$$  \hspace{1cm} (AISC G2-1)

Shear Strength (cont.)

- Nominal Shear Strength $V_n (\varphi_v = 0.9)$
  - plastic when $\lambda \leq \lambda_r$ and $\tau = \tau_r$
  - inelastic when $\lambda \leq \lambda_r$ and $\tau = 0.8\tau_r$
  - elastic when $\lambda > \lambda_r$ and $\tau = \tau_r$

![Graph](image)

Shear Strength (cont.)

- AISC G2 Nominal Shear Strength $V_n$
  - (a) For $\frac{k}{l_w} \leq 1.10 \frac{kE}{F_{yw}}$
    $$C_v = 1.0$$  \hspace{1cm} (AISC G2-3)

  - (a) For $1.10 \frac{kE}{F_{yw}} \leq \frac{k}{l_w} \leq 1.37 \frac{kE}{F_{yw}}$
    $$C_v = \left[ 1.10 \frac{kE}{F_{yw}} \right]$$  \hspace{1cm} (AISC G2-4)

  - (a) For $1.37 \frac{kE}{F_{yw}} \leq \frac{k}{l_w}$
    $$C_v = \left[ 1.51E \left( \frac{k}{l_w} \right)^2 F_{yw} \right]$$  \hspace{1cm} (AISC G2-5)

- Failure Mode
  - The web of a beam or plate girder buckles when the web shear stress exceeds the critical stress.
### Lateral Bracing Design

**AISC Provisions – Stability**

1. For stiffness $\beta_{\text{reqd}}$, 
   
   $$\beta_{\text{reqd}} = 2 \beta_{\text{ideal}}$$

2. For nominal strength $F_{br}$, 
   
   (a) $F_{br} = \beta_{\text{ideal}} (2\Delta_0)$;
   
   (b) $F_{br} = \beta_{\text{ideal}} (0.004L_b)$

Where $\beta_{\text{ideal}} = P_{cr}/L_b$

### Lateral Bracing Design (cont.)

**AISC Provisions – LRFD Stability**

1. Relative bracing  
   
   Required $P_{rb} = \frac{0.008M_{C_d}}{h_o}$

   $\Phi = 0.75$

   Required $\beta_{rb} = \frac{1}{\phi} \left( \frac{4M_{C_d}}{L_o h_o} \right)$

2. Nodal bracing  
   
   Required $P_{rb} = \frac{0.02M_{C_d}}{h_o}$

   Required $\beta_{rb} = \frac{1}{\phi} \left( \frac{10M_{C_d}}{L_o h_o} \right)$

### Lateral Bracing Design (cont.)

**AISC Provisions – LRFD Stability**

1. Relative bracing  
   
   Required $P_{rb} = 0.004P_r$

   $\Phi = 0.75$

   Required $\beta_{rb} = \frac{1}{\phi} \left( \frac{2P_r}{L_b} \right)$

2. Nodal bracing  
   
   Required $P_{rb} = 0.01P_r$

   Required $\beta_{rb} = \frac{1}{\phi} \left( \frac{8P_r}{L_b} \right)$